

WHAT IS CLAIMED IS:

1. A method for an optimal one-shot phase and frequency estimation for timing acquisition for signals transmitted over a communications channel, the method comprising:
 - sampling a preamble comprising a known string of data bits;
 - estimating the sampled preamble (\bar{Y}), the estimated preamble further comprising an estimated amplitude (\hat{A}), an estimated frequency (\hat{f}), and an estimated phase ($\hat{\Phi}$);
 - calculating a cost function ($C(\hat{f}, \hat{\Phi})$) as a function of the estimated frequency (\hat{f}) and the estimated phase ($\hat{\Phi}$);
 - varying at least one of the estimated frequency (\hat{f}) or estimated phase ($\hat{\Phi}$) to calculate a plurality of cost functions; and
 - selecting the cost function ($C(\hat{f}, \hat{\Phi})$) having a minimum value, wherein said cost function having the minimum value is a function of an optimal estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).
2. The method of claim 1, wherein the preamble is sinusoidal.
3. The method of claim 1, wherein the preamble is sampled once for each data bit in the preamble.
4. The method of claim 1, wherein the sampling comprises the following calculation:
$$\bar{X} = [x_0 \cdots x_N] \text{ where } x_k = A \sin\left(\Phi + k \cdot f \cdot \frac{\pi}{2}\right) + n_k, A \text{ is an amplitude value, } \Phi \text{ is a phase value, } f \text{ is a frequency value, and } n_k \text{ is a noise component of a } k^{\text{th}} \text{ sample.}$$
5. The method of claim 1, wherein the estimating the sampled preamble comprises the following calculation:
$$\bar{Y} = [y_0 \cdots y_N] \text{ where } y_k = \hat{A} \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right).$$

1 6. The method of claim 5, wherein the noise component of the sampled preamble has a
2 standard deviation (σ).

1 7. The method of claim 6, wherein the frequency value of the sampled preamble has a
2 normal distribution having a standard deviation(σ_f).

1 8. The method of claim 7, wherein the calculating comprises the following:

2
$$C(\hat{f}, \hat{\Phi}) = \hat{A}^2 \sum_{k=0}^{N-1} \sin^2 \left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2} \right) - 2\hat{A} \sum_{k=0}^{N-1} x_k \sin \left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2} \right) + \frac{\sigma^2 \cdot (\hat{f} - \bar{f})^2}{\sigma_f^2}, \text{ where}$$

3 \bar{f} is a nominal frequency.

1 9. The method of claim 8, wherein each of the plurality of cost functions is calculated
2 with a different frequency value (\hat{f}) and a different phase value ($\hat{\Phi}$).

1 10. The method of claim 9, wherein the plurality of cost functions are calculated
2 substantially simultaneously.

1 11. The method of claim 1, wherein selecting the minimum value cost function further
2 comprises selecting a plurality of first minimum cost functions such that each of the
3 first minimum cost functions has a different estimated frequency (\hat{f}).

1 12. The method of claim 11, wherein selecting the minimum value cost function further
2 comprises selecting a second minimum cost function from the plurality of first
3 minimum cost functions, and wherein the second minimum cost function is a function
4 of an optimal estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).

1 13. The method of claim 1, wherein selecting the minimum value cost function further
2 comprises selecting a plurality of first minimum cost functions such that each of the
3 first minimum cost functions has a different estimated phase ($\hat{\Phi}$).

1 14. The method of claim 13, wherein selecting the minimum value cost function further
2 comprises selecting a second minimum cost function from the plurality of first
3 minimum cost functions, and wherein the second minimum cost function is a function
4 of an optimal estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).

- 1 15. A communications channel for an optimal one-shot phase and frequency-estimation
2 for timing acquisition for signals transmitted over the communications channel, the
3 communications channel comprising:
4 a sampler for sampling a preamble comprising a known string of data bits;
5 a first calculator for estimating the sampled preamble (\bar{Y}), the estimated
6 preamble further comprising an estimated amplitude (\hat{A}), an estimated frequency
7 (\hat{f}), and an estimated phase ($\hat{\Phi}$);
8 a second calculator for calculating a plurality of cost functions ($C(\hat{f}, \hat{\Phi})$) as a
9 function of the estimated frequency (\hat{f}) and the estimated phase ($\hat{\Phi}$) by varying at
10 least one of the estimated frequency (\hat{f}) or estimated phase ($\hat{\Phi}$); and
11 a selector for determining the cost function ($C(\hat{f}, \hat{\Phi})$) having a minimum
12 value, wherein said cost function having the minimum value is a function of an
13 optimal estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).
- 1 16. The communications channel of claim 15, wherein the preamble is sinusoidal.
- 1 17. The communications channel of claim 15, wherein the sampler samples the preamble
2 once for each data bit in the preamble.
- 1 18. The communications channel of claim 15, wherein the sampler samples the preamble
2 in accordance with the following calculation:
3 $\bar{X} = [x_0 \cdots x_N]$ where $x_k = A \sin\left(\Phi + k \cdot f \cdot \frac{\pi}{2}\right) + n_k$, A is an amplitude value, Φ is a
4 phase value, f is a frequency value, and n_k is a noise component of a k^{th} sample.
- 1 19. The communications channel of claim 15, wherein the first calculator estimates the
2 sampled preamble in accordance with the following calculation:
3 $\bar{Y} = [y_0 \cdots y_N]$ where $y_k = \hat{A} \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right)$.
- 1 20. The communications channel of claim 19, wherein the noise component of the
2 sampled preamble has a standard deviation (σ).

1 21. The communications channel of claim 20, wherein the frequency value of the sampled
2 preamble has a normal distribution having a standard deviation(σ_f).

1 22. The communications channel of claim 21, wherein the second calculator calculates
2 the plurality of cost functions in accordance with the following:

3
$$C(\hat{f}, \hat{\Phi}) = \hat{A}^2 \sum_{k=0}^{N-1} \sin^2 \left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2} \right) - 2\hat{A} \sum_{k=0}^{N-1} x_k \sin \left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2} \right) + \frac{\sigma^2 \cdot (\hat{f} - \bar{f})^2}{\sigma_f^2},$$
 where
4 \bar{f} is a nominal frequency.

1 23. The communications channel of claim 22, wherein each of the plurality of cost
2 functions is calculated with a different frequency value (\hat{f}) and a different phase
3 value ($\hat{\Phi}$).

1 24. The communications channel of claim 23, wherein the plurality of cost functions are
2 calculated substantially simultaneously.

1 25. The communications channel of claim 15, wherein the selector determines the
2 minimum value cost function by selecting a plurality of first minimum cost functions
3 such that each of the first minimum cost functions has a different estimated frequency
4 (\hat{f}).

1 26. The communications channel of claim 25, wherein the selector determines the
2 minimum value cost function by selecting a second minimum cost function from the
3 plurality of first minimum cost functions, and wherein the second minimum cost
4 function is a function of an optimal estimated frequency (\hat{f}) and an optimal
5 estimated phase ($\hat{\Phi}$).

1 27. The communications channel of claim 15, wherein the selector determines the
2 minimum value cost function by selecting a plurality of first minimum cost functions
3 such that each of the first minimum cost functions has a different estimated phase
4 ($\hat{\Phi}$).

1 28. The communications channel of claim 27, wherein the selector determines the
2 minimum value cost function by selecting a second minimum cost function from the
3 plurality of first minimum cost functions, and wherein the second minimum cost

- 4 function is a function of an optimal estimated frequency (\hat{f}) and an optimal
5 estimated phase ($\hat{\Phi}$).
- 1 29. A disk drive system for an optimal one-shot phase and frequency estimation for
2 timing acquisition for signals transmitted over a communications channel, the system
3 comprising:
- 4 rotating magnetic media for storing data;
5 a motor for rotating the magnetic media;
6 a recording head for transmitting data;
7 an actuator for positioning the recording head; and
8 a communications channel for communicating data to be stored on or read
9 from the recording media, wherein the communications channel further comprises a
10 sampler for sampling a preamble comprising a known string of data bits, a first
11 calculator for estimating the sampled preamble (\bar{Y}), a second calculator for
12 calculating a plurality of cost functions ($C(\hat{f}, \hat{\Phi})$) as a function of the estimated
13 frequency (\hat{f}) and the estimated phase ($\hat{\Phi}$) by varying at least one of the estimated
14 frequency (\hat{f}) or estimated phase ($\hat{\Phi}$), and a selector for determining the cost
15 function ($C(\hat{f}, \hat{\Phi})$) having a minimum value, wherein said cost function having the
16 minimum value is a function of an optimal estimated frequency (\hat{f}) and an optimal
17 estimated phase ($\hat{\Phi}$), and wherein the estimated preamble further comprises an
18 estimated amplitude (\hat{A}), an estimated frequency (\hat{f}), and an estimated phase ($\hat{\Phi}$).
- 1 30. The system of claim 29, wherein the preamble is sinusoidal.
- 1 31. The system of claim 29, wherein the sampler samples the preamble once for each data
2 bit in the preamble.
- 1 32. The system of claim 29, wherein the sampler samples the preamble in accordance
2 with the following calculation:

3 $\bar{X} = [x_0 \cdots x_N]$ where $x_k = A \sin\left(\Phi + k \cdot f \cdot \frac{\pi}{2}\right) + n_k$, A is an amplitude value, Φ is a
4 phase value, f is a frequency value, and n_k is a noise component of a k^{th} sample.

1 33. The system of claim 29, wherein the first calculator estimates the sampled preamble
2 in accordance with the following calculation:

3
$$\bar{Y} = [y_0 \cdots y_N] \text{ where } y_k = \hat{A} \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right).$$

1 34. The system of claim 33, wherein the noise component of the sampled preamble has a
2 standard deviation (σ).

1 35. The system of claim 34, wherein the frequency value of the sampled preamble has a
2 normal distribution having a standard deviation (σ_f).

1 36. The system of claim 35, wherein the second calculator calculates the cost functions in
2 accordance with the following:

3
$$C(\hat{f}, \hat{\Phi}) = \hat{A}^2 \sum_{k=0}^{N-1} \sin^2\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right) - 2\hat{A} \sum_{k=0}^{N-1} x_k \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right) + \frac{\sigma^2 \cdot (\hat{f} - \bar{f})^2}{\sigma_f^2}, \text{ where}$$

4 \bar{f} is a nominal frequency.

1 37. The system of claim 36, wherein each of the plurality of cost functions is calculated
2 with a different frequency value (\hat{f}) and a different phase value ($\hat{\Phi}$).

1 38. The system of claim 37, wherein the plurality of cost functions are calculated
2 substantially simultaneously.

1 39. The system of claim 29, wherein the selector determines the minimum value cost
2 function by selecting a plurality of first minimum cost functions such that each of the
3 first minimum cost functions has a different estimated frequency (\hat{f}).

1 40. The system of claim 39, wherein the selector determines the minimum value cost
2 function by selecting a second minimum cost function from the plurality of first
3 minimum cost functions, and wherein the second minimum cost function is a function
4 of an optimal estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).

- 1 41. The system of claim 29, wherein the selector determines the cost minimum value
2 function by selecting a plurality of first minimum cost functions such that each of the
3 first minimum cost functions has a different estimated phase ($\hat{\Phi}$).
- 1 42. The system of claim 41, wherein the selector determines the minimum value cost
2 function by selecting a second minimum cost function from the plurality of first
3 minimum cost functions, and wherein the second minimum cost function is a function
4 of an optimal estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).
- 1 43. A communications channel for an optimal one-shot phase and frequency estimation
2 for timing acquisition for signals transmitted over the communications channel, the
3 communications channel comprising:
4 a means for sampling a preamble comprising a known string of data bits;
5 a means for estimating the sampled preamble (\bar{Y}), the estimated preamble
6 further comprising an estimated amplitude (\hat{A}), an estimated frequency (\hat{f}), and an
7 estimated phase ($\hat{\Phi}$);
8 a means for calculating a plurality of cost functions ($C(\hat{f}, \hat{\Phi})$) as a function of
9 the estimated frequency (\hat{f}) and the estimated phase ($\hat{\Phi}$) by varying at least one of
10 the estimated frequency (\hat{f}) or estimated phase ($\hat{\Phi}$); and
11 a means for selecting the cost function ($C(\hat{f}, \hat{\Phi})$) having a minimum value,
12 wherein said cost function having the minimum value is a function of an optimal
13 estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).
- 1 44. The communications channel of claim 43, wherein the preamble is sinusoidal.
- 1 45. The communications channel of claim 43, wherein the preamble is sampled once for
2 each data bit in the preamble.
- 1 46. The communications channel of claim 43, wherein the means for sampling samples
2 the preamble in accordance with the following calculation:

3 $\bar{X} = [x_0 \cdots x_N]$ where $x_k = A \sin\left(\Phi + k \cdot f \cdot \frac{\pi}{2}\right) + n_k$, A is an amplitude value, Φ is a
 4 phase value, f is a frequency value, and n_k is a noise component of a k^{th} sample.

1 47. The communications channel of claim 43, wherein the means for estimating estimates
 2 the sampled preamble in accordance with the following calculation:

3 $\bar{Y} = [y_0 \cdots y_N]$ where $y_k = \hat{A} \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right)$.

1 48. The communications channel of claim 47, wherein the noise component of the
 2 sampled preamble has a standard deviation (σ).

1 49. The communications channel of claim 48, wherein the frequency value of the sampled
 2 preamble has a normal distribution having a standard deviation (σ_f).

1 50. The communications channel of claim 49, wherein the means for calculating
 2 calculates the cost function in accordance with the following:

3 $C(\hat{f}, \hat{\Phi}) = \hat{A}^2 \sum_{k=0}^{N-1} \sin^2\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right) - 2\hat{A} \sum_{k=0}^{N-1} x_k \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right) + \frac{\sigma^2 \cdot (\hat{f} - \bar{f})^2}{\sigma_f^2}$, where

4 \bar{f} is a nominal frequency.

1 51. The communications channel of claim 50, wherein each of the plurality of cost
 2 functions is calculated with a different frequency value (\hat{f}) and a different phase
 3 value ($\hat{\Phi}$).

1 52. The communications channel of claim 51, wherein the plurality of cost functions are
 2 calculated substantially simultaneously.

1 53. The communications channel of claim 43, wherein means for selecting selects the
 2 minimum value cost function by selecting a plurality of first minimum cost functions
 3 such that each of the first minimum cost functions has a different estimated frequency
 4 (\hat{f}).

- 1 54. The communications channel of claim 53, wherein the means for selecting selects the
2 minimum value cost function by selecting a second minimum cost function from the
3 plurality of first minimum cost functions, and wherein the second minimum cost
4 function is a function of an optimal estimated frequency (\hat{f}) and an optimal
5 estimated phase ($\hat{\Phi}$).
- 1 55. The communications channel of claim 43, wherein the means for selecting selects the
2 minimum value cost function by selecting a plurality of first minimum cost functions
3 such that each of the first minimum cost functions has a different estimated phase
4 ($\hat{\Phi}$).
- 1 56. The communications channel of claim 55, wherein the means for selecting selects the
2 minimum value cost function by selecting a second minimum cost function from the
3 plurality of first minimum cost functions, and wherein the second minimum cost
4 function is a function of an optimal estimated frequency (\hat{f}) and an optimal
5 estimated phase ($\hat{\Phi}$).
- 1 57. A computer program product containing a program for providing an optimal one-shot
2 phase and frequency estimation for timing acquisition for signals transmitted over a
3 communications channel, the program comprising:
4 sampling a preamble comprising a known string of data bits;
5 estimating the sampled preamble (\bar{Y}), the estimated preamble further
6 comprising an estimated amplitude (\hat{A}), an estimated frequency (\hat{f}), and an
7 estimated phase ($\hat{\Phi}$);
8 calculating a cost function ($C(\hat{f}, \hat{\Phi})$) as a function of the estimated frequency
9 (\hat{f}) and the estimated phase ($\hat{\Phi}$);
10 varying at least one of the estimated frequency (\hat{f}) or estimated phase ($\hat{\Phi}$) to
11 calculate a plurality of cost functions; and

12 selecting the cost function ($C(\hat{f}, \hat{\Phi})$) having a minimum value, wherein said
13 cost function having the minimum value is a function of an optimal estimated
14 frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).

1 58. The computer program product of claim 57, wherein the preamble is sinusoidal.

1 59. The computer program product of claim 57, wherein the preamble is sampled once for
2 each data bit in the preamble.

1 60. The computer program product of claim 57, wherein the sampling comprises the
2 following calculation:

3 $\bar{X} = [x_0 \cdots x_N]$ where $x_k = A \sin\left(\Phi + k \cdot f \cdot \frac{\pi}{2}\right) + n_k$, A is an amplitude value, Φ is a
4 phase value, f is a frequency value, and n_k is a noise component of a k^{th} sample.

1 61. The computer program product of claim 57, wherein the estimating the sampled
2 preamble comprises the following calculation:

3 $\bar{Y} = [y_0 \cdots y_N]$ where $y_k = \hat{A} \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right)$.

1 62. The computer program product of claim 51, wherein the noise component of the
2 sampled preamble has a standard deviation (σ).

1 63. The computer program product of claim 62, wherein the frequency value of the
2 sampled preamble has a normal distribution having a standard deviation (σ_f).

1 64. The computer program product of claim 63, wherein the calculating comprises the
2 following:

3 $C(\hat{f}, \hat{\Phi}) = \hat{A}^2 \sum_{k=0}^{N-1} \sin^2\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right) - 2\hat{A} \sum_{k=0}^{N-1} x_k \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right) + \frac{\sigma^2 \cdot (\hat{f} - \bar{f})^2}{\sigma_f^2}$, where

4 \bar{f} is a nominal frequency.

1 65. The computer program product of claim 64, wherein each of the plurality of cost
2 functions is calculated with a different frequency value (\hat{f}) and a different phase
3 value ($\hat{\Phi}$).

- 1 66. The computer program product of claim 65, wherein the plurality of cost functions are
2 calculated substantially simultaneously.
- 1 67. The computer program product of claim 57, wherein selecting the minimum value
2 cost function further comprises selecting a plurality of first minimum cost functions
3 such that each of the first minimum cost functions has a different estimated frequency
4 (\hat{f}).
- 1 68. The computer program product of claim 67, wherein selecting the minimum value
2 cost function further comprises selecting a second minimum cost function from the
3 plurality of first minimum cost functions, and wherein the second minimum cost
4 function is a function of an optimal estimated frequency (\hat{f}) and an optimal
5 estimated phase ($\hat{\Phi}$).
- 1 69. The computer program product of claim 57, wherein selecting the minimum value
2 cost function further comprises selecting a plurality of first minimum cost functions
3 such that each of the first minimum cost functions has a different estimated phase
4 ($\hat{\Phi}$).
- 1 70. The computer program product of claim 69, wherein selecting the minimum value
2 cost function further comprises selecting a second minimum cost function from the
3 plurality of first minimum cost functions, and wherein the second minimum cost
4 function is a function of an optimal estimated frequency (\hat{f}) and an optimal
5 estimated phase ($\hat{\Phi}$).
- 1 71. A disk drive system for an optimal one-shot phase and frequency estimation for
2 timing acquisition for signals transmitted over a communications channel, the system
3 comprising:
4 means for storing data;
5 means for rotating the means for storing;
6 means for transmitting data to and from the means for storing;
7 means for positioning the means for transmitting data; and
8 means for communicating data to be stored on or read from the means for
9 storing, wherein said means for communicating further comprises means for sampling.

10 a preamble comprising a known string of data bits, means for estimating the sampled
11 preamble (\bar{Y}), means for calculating a plurality of cost functions ($C(\hat{f}, \hat{\Phi})$) as a
12 function of the estimated frequency (\hat{f}) and the estimated phase ($\hat{\Phi}$) by varying at
13 least one of the estimated frequency (\hat{f}) or estimated phase ($\hat{\Phi}$), and means for
14 determining the cost function ($C(\hat{f}, \hat{\Phi})$) having a minimum value, wherein said cost
15 function having the minimum value is a function of an optimal estimated frequency
16 (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$), and wherein the estimated preamble further
17 comprises an estimated amplitude (\hat{A}), an estimated frequency (\hat{f}), and an estimated
18 phase ($\hat{\Phi}$)

1 72. The system of claim 71, wherein the preamble is sinusoidal.

1 73. The system of claim 71, wherein the means for sampling samples the preamble once
2 for each data bit in the preamble.

1 74. The system of claim 71, wherein the means for sampling samples the preamble in
2 accordance with the following calculation:

3 $\bar{X} = [x_0 \cdots x_N]$ where $x_k = A \sin\left(\Phi + k \cdot f \cdot \frac{\pi}{2}\right) + n_k$, A is an amplitude value, Φ is a
4 phase value, f is a frequency value, and n_k is a noise component of a k^{th} sample.

1 75. The system of claim 71, wherein the means for estimating the sampled preamble in
2 accordance with the following calculation:

3 $\bar{Y} = [y_0 \cdots y_N]$ where $y_k = \hat{A} \sin\left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2}\right)$.

1 76. The system of claim 75, wherein the noise component of the sampled preamble has a
2 standard deviation (σ).

1 77. The system of claim 76, wherein the frequency value of the sampled preamble has a
2 normal distribution having a standard deviation (σ_f).

1 78. The system of claim 77, wherein the means for calculating calculates the cost
2 functions in accordance with the following:

3
$$C(\hat{f}, \hat{\Phi}) = \hat{A}^2 \sum_{k=0}^{N-1} \sin^2 \left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2} \right) - 2\hat{A} \sum_{k=0}^{N-1} x_k \sin \left(\hat{\Phi} + k \cdot \hat{f} \cdot \frac{\pi}{2} \right) + \frac{\sigma^2 \cdot (\hat{f} - \bar{f})^2}{\sigma_f^2}, \text{ where}$$

4 \bar{f} is a nominal frequency.

1 79. The system of claim 78, wherein each of the plurality of cost functions is calculated
2 with a different frequency value (\hat{f}) and a different phase value ($\hat{\Phi}$).

1 80. The system of claim 79, wherein the plurality of cost functions are calculated
2 substantially simultaneously.

1 81. The system of claim 71, wherein the means for selecting determines the minimum
2 value cost function by selecting a plurality of first minimum cost functions such that
3 each of the first minimum cost functions has a different estimated frequency (\hat{f}).

1 82. The system of claim 81, wherein the means for selecting determines the minimum
2 value cost function by selecting a second minimum cost function from the plurality of
3 first minimum cost functions, and wherein the second minimum cost function is a
4 function of an optimal estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).

1 83. The system of claim 71, wherein the means for selecting determines the cost
2 minimum value function by selecting a plurality of first minimum cost functions such
3 that each of the first minimum cost functions has a different estimated phase ($\hat{\Phi}$).

1 84. The system of claim 83, wherein the means for selecting determines the minimum
2 value cost function by selecting a second minimum cost function from the plurality of
3 first minimum cost functions, and wherein the second minimum cost function is a
4 function of an optimal estimated frequency (\hat{f}) and an optimal estimated phase ($\hat{\Phi}$).

1